Derivation of an expression for X in terms of VSWR for a normalised load where R is constant and X changes with frequency

Owen Duffy

Abstract

This article presents a derivation of the 'classic' VSWR curve as exhibited by many antenna types.

General expression for complex reflection coefficient, Γ:

$$\Gamma = \frac{Zl - Zo}{Zl + Zo}$$

Magnitude of complex reflection coefficient:

$$\rho = |\Gamma|$$

Where r=R/Zo=1 and x=X/R:

$$\rho = \left|\frac{1 + jx - 1}{1 + jx + 1}\right|$$

$$\rho = \frac{|x|}{\sqrt{x^2 + 4}}$$

VSWR v in terms of ρ :

$$v = \frac{1 + \rho}{1 - \rho}$$

VSWR v in terms of x :

$$v = \frac{1 + \frac{|x|}{\sqrt{x^2 + 4}}}{1 - \frac{|x|}{\sqrt{x^2 + 4}}}$$

Multiplying top and bottom by $\sqrt{x^2 + 4}$:

$$v = \frac{\sqrt{x^2 + 4} + |x|}{\sqrt{x^2 + 4} - |x|}$$

Multiplying top and bottom by $\sqrt{x^2 + 4} + |x|$:

$$v = \left(\frac{\sqrt{x^2 + 4} + |x|}{2}\right)^2$$

Taking the square root of both sides:

$$\sqrt{v} = \frac{\sqrt{x^2 + 4} + |x|}{2}$$

Subtract $\frac{1}{\sqrt{v}}$ from both sides:

$$\sqrt{v} - \frac{1}{\sqrt{v}} = \frac{\sqrt{x^2 + 4} + |x|}{2} - \frac{2}{\sqrt{x^2 + 4} + |x|}$$

Multiply last term top and bottom by $\sqrt{x^2 + 4} - |x|$:

$$\sqrt{v} - \frac{1}{\sqrt{v}} = \frac{\sqrt{x^2 + 4} + |x|}{2} - \frac{2\left(\sqrt{x^2 + 4} - |x|\right)}{4}$$

$$\sqrt{v} - \frac{1}{\sqrt{v}} = \frac{2|x|}{2}$$

$$\sqrt{v} - \frac{1}{\sqrt{v}} = |x|$$

$$|x| = \frac{v - 1}{\sqrt{v}}$$

QED

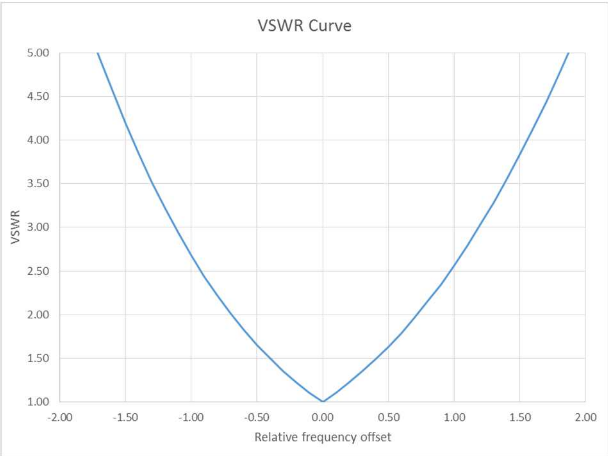

Figure 1: Form of the classic VSWR curve
